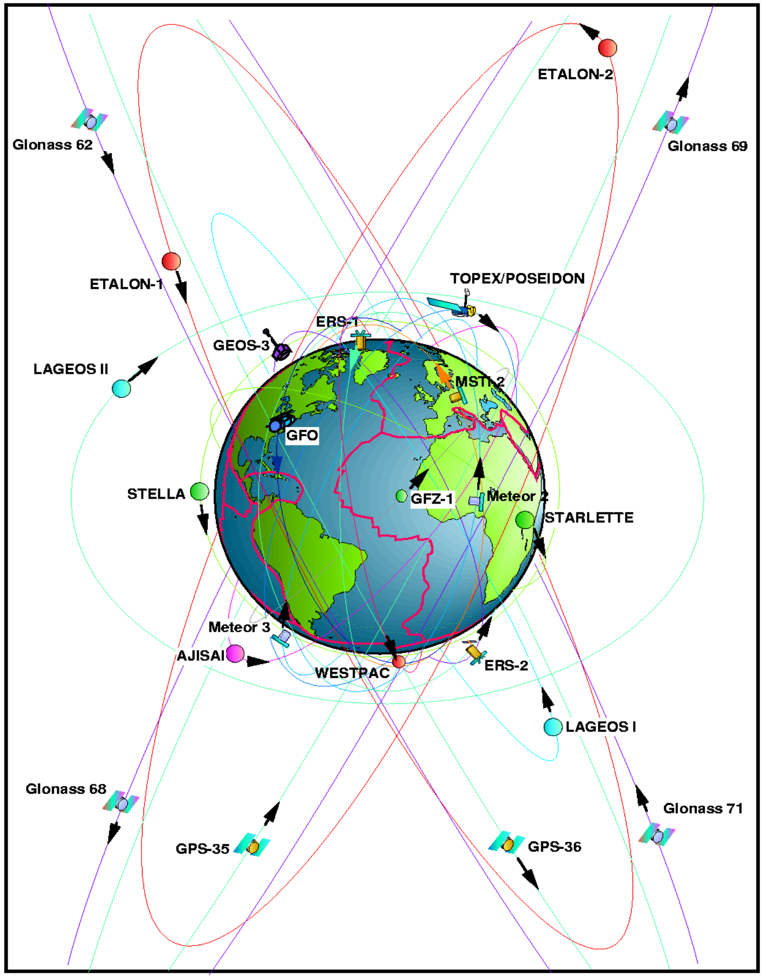
Satellite Laser Ranging and the Terrestrial Reference Frame; Principal Sources of Uncertainty in the Determination of the Scale



J.C. Ries

Center for Space Research
The University of Texas at Austin

EGU General Assembly 2007
Vienna
April 16-20, 2007

Why do we use different techniques?

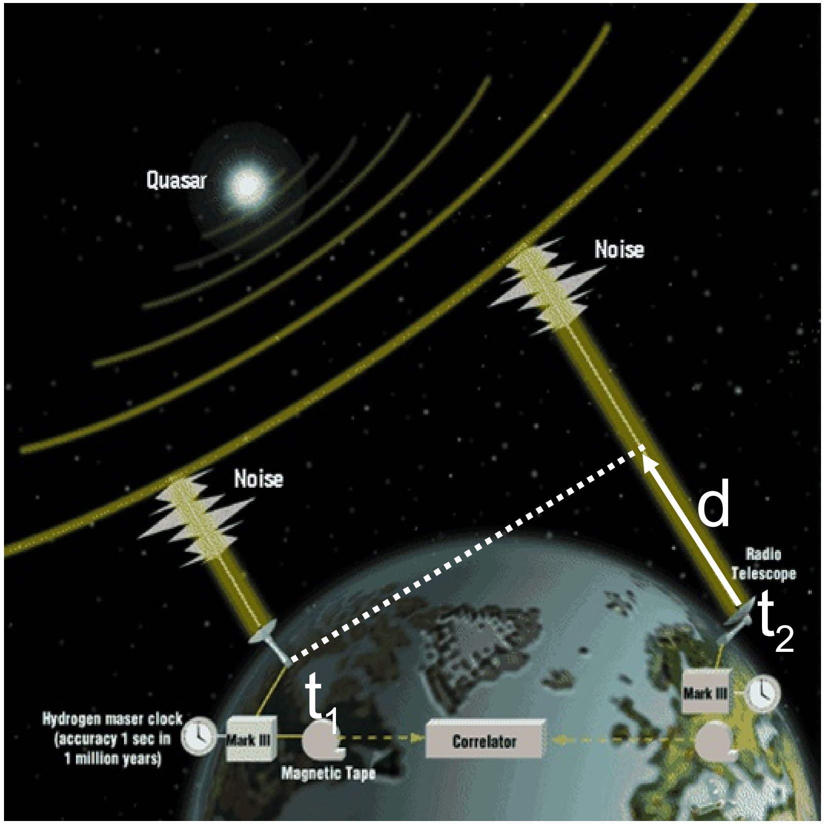
- Each technique has fundamentally different observations with unique contributions to the TRF
- Where they overlap, they can provide cross validation and increased accuracy (or uncover a discrepancy)
- The current precision for determining the TRF scale is sufficient that a discrepancy between SLR and VLBI at the ppb level is probably significant (our concern here)

TECHNIQUE **Signal** **Source** **Obs. Type**	**VLBI** **Microwave** **Quasars** **Time difference**	**SLR** **Optical** **Satellite** **Two-way absolute range**	**GPS/DORIS** **Microwave** **Satellites** **Range change**
Celestial Frame UT1	**Yes**	No	No
Polar Motion	**Yes**	Yes	**Yes**/Yes
Scale (absolute lengths)	**Yes**	**Yes**	Yes
Geocenter (origin)	No	**Yes**	Yes
Geographic Density	No	No	**Yes**

Yes means contributed to ITRF2005
Yes means has strong capability but not used
Yes means some capability exists

Scale & Terrestrial Reference Frame (1)

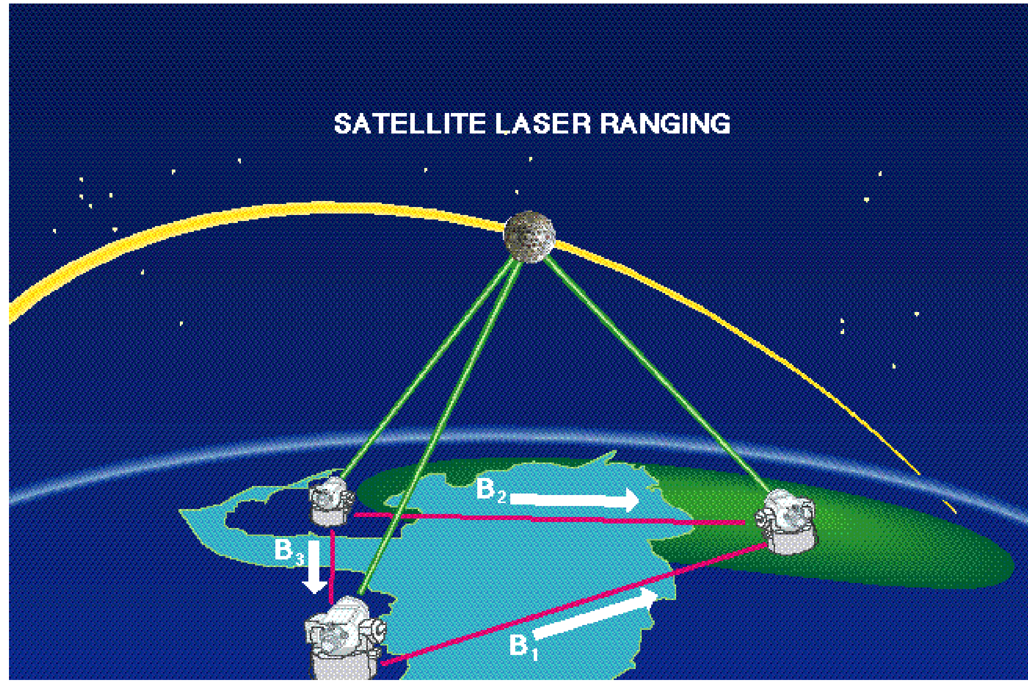
- A scale change is the uniform increase or decrease of all distances
  - 1 ppb change in scale ⇒ ~6 mm change in station height
- VLBI determines the distance vectors kinematically
- There are no dynamics involved and there is no connection to Earth's mass center
- Earth's mass enters only through a relativistic time delay correction



$$d = c\,(t_2 - t_1) + \text{corrections}$$

Scale & Terrestrial Reference Frame (2)

- SLR measures station location indirectly through an intermediate target
- Satellite orbital period is related to orbital radius and Earth's mass (GM)
- Laser ranging provides absolute orbital height and curvature, so we can estimate orbit, station heights, biases and GM simultaneously
- In light of the scale issue in the recent ITRF2005 combination, was there be a problem in the background model for SLR that could be biasing the TRF results?



GM estimation from SLR (1)

- In 1992, GM estimated using 5 years of LAGEOS-1 data to determine value currently still in use
  - GM = 398600.4415 ± 0.0008 km^3/s^2 (TDT value)
    - GM (SI) = GM (TDT) * $(1+L_G)$ = 398600.4418
  - Considered 2 cm biases plus a 'quesstimate' for troposphere error (0.2% or ~4-5 mm in zenith delay)
    - Possible systematic error in Murini & Murray model was a concern
    - Did not consider contribution of Center of Mass (CoM) offset errors
- In 2005, estimated GM using 12+ years of SLR data from LAGEOS and LAGEOS-2
  - GM = 398600.44163 ± 0.00042 km^3/s^2 (± 1 ppb)
  - 'Formal' error = 0.00004 km^3/s^2 (0.1 ppb)
    - 'Formal' error estimate already includes 4-6X increase in the apriori SLR data standard deviation to try to better reflect systematic errors

GM estimation from SLR (2)

- Considering a 1 cm bias (single average bias) for each station increased uncertainty to 0.00027 (~ 0.7 ppb)
  - Estimating or not estimating biases changed the GM solution by less than the estimated uncertainty
- Atmosphere refraction contribution estimated to be < 0.2 ppb
  - Compared estimates using Mendes & Pavlis refraction model to standard Marini & Murray model
  - Difference in GM was only 0.3 ppb, about the same size as the difference between the LAGEOS and LAGEOS-2 estimates
  - Assuming most of the difference is error in the older M&M model, refraction errors might be assumed to contribute no more than 0.1-0.2 ppb to the uncertainty in GM (or scale)

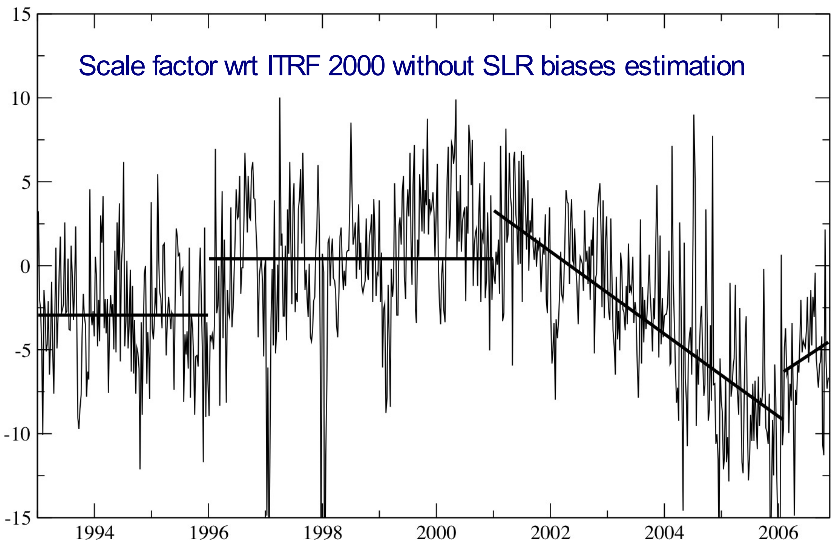
GM estimation from SLR (3)

- CoM model was identified as likely ‘tall pole’ in error budget
  - Using ‘guesstimate’ of 4 mm error in the CoM correction led to an increase of the estimated error in GM to 0.00042 (~1 ppb)
  - ITRF2005 scale issue motivated more careful analysis of impact of CoM model errors on GM, for LAGEOS and other satellites

Satellite (A in Earth radii)	CoM Error required for 1 ppb error in GM
Starlette (~1)	1 mm
LAGEOS (~2)	3 mm
GPS (~4)	~8 mm (extrapolated)

- Low satellites are much too sensitive to CoM errors
- Laser reflectors on high-altitude satellites, such as future GPS satellites, could provide helpful scale information but the CoM has to be known very well to provide better accuracy

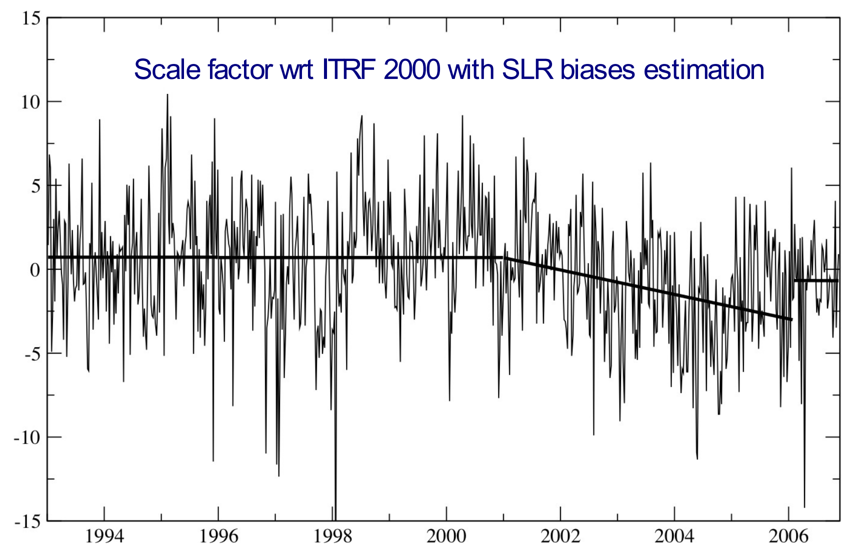
Impact of biases on TRF determination



Coulot, D., P. Berio, D. Féraudy, O. Laurain, and P. Exertier, *Different ways of considering biases for Satellite Laser Ranging data processing: consequences on Terrestrial Reference Frame scale factors, submitted to Geophys. Res. Lett., 2007.* ***(see also poster in G1 session)***

SLR ranging biases should be routinely estimated (with appropriate care)

Without a ‘place to go’, biases will distort TRF determination



General Relativity and the TRF scale (1)

- For SLR, we have adopted a terrestrial time coordinate (TT) that differs from geocentric coordinate time (TCG) by the influence of the Earth's gravitational potential on time as measured on the Earth's surface
  - $d(TT)/d(TCG) = (1-L_G)$ where $L_G = U/c^2 \approx 0.7 \times 10^{-9}$ $(U=GM/R_e)$
  - Since the speed of light is a defined quantity, the result is a unit of length that is also scaled by 0.7 ppb
  - Our distances differ from 'physical distances' by 0.7 ppb, but this concept is meaningless for intercontinental length scales and has no practical consequence (as long as we all agree)
- For VLBI, the 'consensus' model was adopted to be consistent with the SLR realization of the geocentric frame

General Relativity and the TRF scale (2)

This equation is converted into a geocentric delay equation using known quantities by performing the relativistic transformations relating the barycentric vectors $\vec{X}_i$ to the corresponding geocentric vectors $\vec{x}_i$, thus converting equation 8 into an equation in terms of $\vec{x}_i$. The related transformation between barycentric and geocentric time can be used to derive another equation relating $T_2 - T_1$ and $t_2 - t_1$, and these two equations can then be solved for the geocentric delay in terms of the geocentric baseline vector $\vec{b}$. In the rational polynomial form the total geocentric vacuum delay is given by

$$t_{v_2} - t_{v_1} = \frac{\Delta T_{grav} - \frac{\hat{K}\cdot\vec{b}}{c}\left[1 - \frac{(1+\gamma)U}{c^2} - \frac{|\vec{V}_\oplus|^2}{2c^2} - \frac{\vec{V}_\oplus\cdot\vec{w}_2}{c^2}\right] - \frac{\vec{V}_\oplus\cdot\vec{b}}{c^2}(1 + \hat{K}\cdot\vec{V}_\oplus/2c)}{1 + \frac{\hat{K}\cdot(\vec{V}_\oplus + \vec{w}_2)}{c}}. \qquad (9)$$

(IERS 2003 standards)

- To realize the same reference frame as SLR, the potential term excludes the potential due to the Earth, but it is multiplied by a factor of 2 (γ=1 in GR)
- VLBI distances are smaller than 'physical' distances by 1.4 ppb; SLR distances are smaller by only 0.7 ppb
- It would appear that they are not defining the same reference frame

Are we missing anything else?

- Ashby (2003) proposed that, due to the time definition, an additional term (of 0.7 ppb) is required for the light-time correction (for GPS but it would be true for SLR and DORIS)

> The time delay is so small that quadrupole contributions to the potential (and to Φ_0) can be neglected. Integrating along the straight line path a distance l between the transmitter and receiver gives for the time delay
>
> $$\Delta t_{delay} = \frac{\Phi_0}{c^2}\frac{l}{c} + \frac{2GM}{c^3}\ln\left[\frac{r_1 + r_2 + l}{r_1 + r_2 - l}\right] \quad (53)$$
>
> The second term is the usual expression for the Shapiro time delay. It is modified for GPS by a term of opposite sign (Φ_0 is negative), due to the choice of coordinate time rate, which tends to cancel the logarithm term.

- This correction is intriguing because it appears to reduce the bias between SLR and VLBI; whether it is correct is unclear

Conclusions

- Biases in range data need not bias scale, since these can be included in the estimation (with appropriate constraints)
  - Apparent SLR scale variations in SLR contribution to ITRF2005 may be explainable by unmodeled/unestimated biases
- Center-of-Mass offset model for LAGEOS satellites directly determines estimate for GM $\Rightarrow$ TRF scale
  - 3 mm change in CoM for all stations would change GM estimate by 1 ppb (6 mm change already adopted for one site)
- Is GM a defining a constant or a parameter that we continue to refine, even if the result is to rescale all satellite orbits?
  - Changes to altimeter satellite orbit heights at the mm-level would impact global mean sea level time series
- Are there discrepancies in the application of General Relativity to the SLR or VLBI techniques?